CS 526
Advanced Compiler Construction

https://charithm.web.illinois.edu/cs526/sp2022/
(slides adapted from Sasa and Vikram)
INTERPROCEDURAL ANALYSIS

The slides adapted from Vikram Adve
So Far…

Control Flow Analysis

Data Flow Analysis

Dependence Analysis

Points-to Analysis

All within a single procedure (intraprocedural)
Today

Control Flow Analysis

Data Flow Analysis

Dependence Analysis

Points-to Analysis

Across multiple procedures (interprocedural)
Today

Control Flow Analysis

Key question to answer:
How to deal with function call $y = f(x)$?

(we will describe this for a subset of techniques)
Why interprocedural analysis and optimization?

• **Produce better code around call sites**
  avoid saves, restores; understand cross-call site data flow

• **Produce tailored copies of procedures**
  often, full generality is not necessary; constant valued parameters, aliases

• **Provide sharper global (*intraprocedural*) analysis**
  improve on conservative assumptions especially true for global variables

• **Present the optimizer with more context**
  languages with short procedures; assumes context improves code
Key Challenges

Compilation Time, Memory
Key problem: scalability to large programs
• Dominated by analysis time/memory
• Flow-sensitive analyses: bottleneck often memory (!time)
• ⇒ Often limited to fast but imprecise analyses

Multiple calling environments
Different calls to P() have different properties:
• known constants, aliases, surrounding execution context (e.g., enclosing loops), function-pointer arguments, …
• frequency of the call
Key Challenges

Recursion
Recursive codes are typically like most difficult types of loops
• No induction variables, complex data structures, complex termination

Estimating profitability
• even inlining is not clear win
• separation of concerns:
  • ignores resource constraints
  • works best with smaller procedures
Solution #1: Reduction to Intraprocedural

1. **Conservative:**
   - Analyze each function separately
   - At every function call, invalidate all global variables
   - The result for each function is conservative, for all values of the input variables

2. **Inlining:**
   - At each call, insert the function body
   - Can optimize better, use local values of variables
   - However, the control flow graph grows exponentially
   - Also, recursion causes problems
Inlining Benefits

.performance improvement (%)

An Experiment with Inline Substitution, Cooper et al. 1991
Solution #2: Analyze Global Flows

Create Whole-Program CFG

- Possible unrealizable paths
- Tradeoff between precision and space

Call String Approach

- Maintain the context of caller, each call site can have a different analysis
- Call context simulates stack
- Finite unrolling for recursion
Realizable Paths

Definition: Realizable Path
A program path is realizable iff every procedure call on the path returns control to the point where it was called (or to a legal exception handler or program exit)

Whole-program Control Flow Graph?
Conceptually extend CFG to span whole program:
- split a call node in CFG into two nodes: CALL and RETURN
- add edge from CALL to ENTRY node of each callee
- add edge from EXIT node of each callee to RETURN
Problem: This produces many unrealizable paths

Focusing only on realizable paths requires context-sensitive analysis
MOP and MVP Solutions

Previously, we learned about meet-over-paths (MOP) solutions for dataflow equations

- These were desired solutions of the analysis

For interprocedural analysis, we need to define a new meet-over-valid-paths (MVP) solution, which only combines dataflow facts over the realizable paths.

- Avoids the paths induced by conservative whole-program CFG.
- These would be the desired solutions of interprocedural problems
Call Graph

Call Graph:

• represents how the procedures (subprograms) are being called within the program code

• Nodes represent procedures, e.g., f, g…

• Edges (f, g) specify the caller and the callee, e.g., procedure f calls procedure g.

• A cycle in the graph indicates recursive procedure calls
Building the Call Graph

Function pointer variables make this problem hard!
Fortran: only formal arguments (no assignment)
C, C++, Java, ...: arbitrary function pointer variables and uses

```c
void main () {
    confuse(a,c)
    confuse(b,d)
}

void confuse(fptr1 x, fptr0 y) { (*x)(y) }
void a(fptr0 z) { (*z)() }
void b(fptr0 z) { (*z)() }
void c { ... }
void d { ... }
```
Languages with Function Pointer Assignment

Approach 1: Solve CALLS and ALIAS separately
- Compute whole-program call graph
- Solve ALIAS
- Refine call graph
  (Iterate ALIAS and CALLS until there are no changes)

Approach 2: Solve CALLS and ALIAS simultaneously
Context-sensitive alias analysis algorithms can discover call graph as they propagate points-to sets:
- Liang and Harrold (FSE 1999)
- Fähndrich, Rehof and Das (PLDI 2000)
- Lattner and Adve (PLDI 2007)
Call Graph: Previous Results

Fortran with Recursion
Precise graph: Callahan, Carle, Hall, Kennedy (87, 90)
- \(O(N^{v_{\text{max}}+1})\) logical steps \(N = \#\text{procedures}\)
  \(v_{\text{max}} = \text{max.} \#\text{procedure-valued parameters for any procedure}\)
Conservative, approximate graph: Hall, Kennedy (90)
- \(O(N + PE)\) logical steps \(P = \#\text{procedures passed as parameters}\)

Object-oriented Languages
A framework for call graph construction algorithms, David Grove, Craig Chambers. *ACM TOPLAS*, 23(6), November 2001
- Describes several alternative algorithms in a common framework
- Incorporates class hierarchy analysis, MOD, exception analysis, escape analysis
Solution #3: Functional Approach

**Previous:** Saves space, but still iterates many times of the function

**Goal:** Establish the input/output relationship for the function, i.e., compute function summary

- Analyze once, compute function summary
- At call sites, specialize this summary, without looking at the body
- For recursive calls, unroll
Classification of IP* Analyses

Flow-insensitive: computes a single result for entire program/procedure
• Can be solved in time polynomial in the size of the call graph (Banning, POPL, 1979)

Flow-sensitive: computes distinct result for each program point
• NP-complete or Co-NP complete (Myers, POPL, 1981).

Context-insensitive: includes realizable and unrealizable paths

Context-sensitive: explicitly excludes unrealizable paths

May problems describe events that may happen as the result of executing a given call

Must problems describe events that always happen when a given call is executed

IP* = Interprocedural
Classical IP problems

Side-effect problems: “backward” IP dataflow problems
Propagation problems: “forward” IP dataflow problems
(where backward and forward refer to call-graph).

- **CALLS**: Constructing the call graph
- **ALIAS**: Alias analysis
- **MOD**: Variables possibly modified due to a call
- **REF**: Variables possibly used due to a call
- **KILL**: Variables definitely modified before use due to a call
- **USE**: Variables possibly used before being modified due to a call
- **CONST**: Constant propagation
IP Constant Propagation

The problem
Compute sets of pairs \((name, value)\) at entry to each function and after each call site, where \(value\) is an element of the usual CONST lattice \((\top, \bot, \text{or constant value})\).

Key considerations
1. Constant values available at call sites
   - deriving initial information
2. Transmission of values across call sites and returns
   - interprocedural data-flow problem
3. Transmission of values through procedure bodies
   - single procedure data flow \((\text{jump function})\)
IP Constant Propagation

Build interprocedural value graph

• analogous to the SSA graph used in SCCP
• standard CONST lattice: values are either $\top$, (constant), or $\bot$

Use a standard iterative approach:

• maintain a worklist of formal parameters
• add a parameter to the worklist every time it changes value
• any parameter changes value at most twice
IP Constant Propagation

Challenges:
1. Overall problem is undecidable.
2. Constant propagation is flow-sensitive:
   ⇒ Must have all procedures in memory simultaneously

Solution: Capture approximate effects of function bodies with “jump functions.”

Callahan, Cooper, Kennedy, and Torczon, “Interprocedural constant propagation”, SIGPLAN 86, July 1986.

IP Constant Propagation

Use two types of jump functions:

- **forward jump function**: value passed to a formal parameter at a call-site (as function of formal parameters of caller)

- **return jump function**: each return value from a procedure (as a function of formal parameters of the procedure)

For a procedure p we define $J_s^y$ - for an actual parameter y gives the expression of p’s formal arguments at the call site s
Example Jump Functions

Literal Constant Jump Function:
\[ J^y_s = c, \text{ if } y \text{ is the literal constant } c \text{ at call site } s \] (else, $\perp$)

Intraprocedural Constant Jump Function:
\[ J^y_s = c, \text{ if intraprocedural analysis or value numbering can prove } y = c \text{ at the call site } s \] (else, $\perp$)

Pass-through Parameter Jump Function:
\[ J^y_s = c, \text{ (as above), or } x, \text{ if } y = x \text{ at } s \text{ and } x \text{ is a formal parameter of the calling procedure} \] (else, $\perp$)

Polynomial Parameter Jump Function:
\[ J^y_s = c, \text{ (as above), or } f(\hat{x}), \text{ if } y = f(\hat{x}) \text{ at } s, \text{ where } \hat{x} \text{ are formal parameters of the calling procedure and } f \text{ is a polynomial function} \] (else, $\perp$)
## Constants found through the use of jump functions


<table>
<thead>
<tr>
<th>Program</th>
<th>Using Return Jump Functions</th>
<th>No Return Jump Functions</th>
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<tbody>
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Compilers rock!! Congratulations!!

BREAKING: This year’s $1 millionn Turing Award - often described as "the Nobel Prize for computing" - goes to Jeffrey Ullman & Alfred Aho for their work in compilers.

They co-wrote 2 classic computer science texts: the green and red "dragon books" (1977 & 1986).

More info: https://www.cnet.com/.../turing-award-goes-to.../
Interprocedural Side-Effect Problems


Problems (for a call site s: y = f(x1…xn) )

• **MOD(s):**
  \[ v \in \text{MOD}(s) \iff \text{statement } s \text{ may change value of variable } v \]

• **MOD(P):**
  \[ v \in \text{MOD}(F) \iff \text{function } F \text{ may change value of variable } v \]

• Similarly **REF(s), REF(F):**
  \[ v \in \text{REF}(\ast) \iff \text{statement/function might reference } v \text{'s value} \]
Interprocedural Side-Effect Analysis

Compute: $\text{MOD}(s), \text{MOD}(F), \text{REF}(s), \text{REF}(F)$

Strategy

1. Perform interprocedural alias analysis (perhaps context-sensitive)

2. Compute direct side-effects of assignments

3. Solve dataflow equations iteratively on the Interprocedural Control Flow Graph
   - Use context in each dataflow equation
   - Here context captured by reaching aliases – \textbf{RAs}

(see: Landi and Ryder. A safe approximation algorithm for interprocedural pointer aliasing. PLDI 1992)
Reaching Alias

The data-flow fact that x and y are aliased at program point $n$ is represented by an unordered pair $<x, y>$ at $n$. The encoding of calling context is the set of reaching aliases (RAs) that exists at entry of procedure $p$ containing $n$ when $p$ is invoked from a particular call site.

---

```c
int *p, q, r;
void main ()
{
    p = &q;
    n1 : A ();
    p = &r;
    n2 : A ();
}
void A ()
{
    B ();
    n3 : B ();
}
void B ()
{
}
```

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<td>$[\phi, (*p, r)]$</td>
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<td>$[(*p, q), (*p, q)]$, $[(*p, r), (*p, r)]$</td>
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Interprocedural Side-Effect Analysis

Assumptions:

• Simple programs
• No setjmp and longjmp
• “By-reference” passing: pointers
Example

```c
int x, y, k;
R(int *b)
{
    if (*b)
    {
        b = &k;
        *b = 0;
    }
    (*b)++
}

main()
{
    R(&x);
    R(&y);
}
```
```c
int x, y, k;
R(int *b)
{
    if (*b)
    {
        b = &k;
        *b = 0;
    }
    (*b)++
}

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    R(&x);
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```
Decomposition of the Analysis MOD(n) and MOD(P)

P – Procedure
RA – Calling Context (Reaching Aliases)
n – Program point (statement)

MOD(n) variables modified by statement n, summarizing all contexts

CMOD(n, RA) variables modified by statement n under RA

MOD(P) variables modified by procedure P, summarizing all contexts

PMOD(P, RA) variables modified by procedure P under RA

CondLIMOD(P, RA) variables modified by assignments in procedure P, under context RA

CondLMOD(n, RA) variables modified by assignment n due to aliases after any predecessor of n

Alias Analysis in context RA

Alias(n, RA)

DIRMOD(n) variables directly modified by assignment n
int x, y, k;
R(int *b)
{
  if (*b)
  {
    b = &k;
    *b = 0;
  }
  (*b)++
}
main()
{
  R(&x);
  R(&y);
}

\[ \text{Example} \]

### Decomposition of the Analysis

- **MOD(n)**: Program point (statement)
- **MOD(P)**: Procedure
- **RA**: Calling Context (Reaching Aliases)
- **PMOD(P, RA)**: Procedure P under RA
- **CMOD(n, RA)**: Variables modified by statement n under RA
- **DIRMOD(n, RA)**: Variables directly modified by assignment n
- **CondMOD(n, RA)**: Variables modified by assignments in procedure P under context RA
- **CondIMOD(P, RA)**: Variables modified by procedure P under RA

### Flow Sensitive Analysis Results

Global variables in C are initialized to zero

\[ ^\text{Flow sensitive analysis results} \]
Example

Decomposition of the Analysis

\[ \text{MOD}(n) \text{ and } \text{MOD}(P) \]

P - Procedure
RA - Calling Context (Reaching Aliases)
\( n \) - Program point (statement)

- MOD(n) \( n \) summarizing all contexts
- MOD(P, RA) \( P \) under context RA
- MODIMOD(n, RA) \( n \) modified by procedure and assignments in procedure
- ConLMOD(n, RA) \( n \) modified by assignments due to aliases after any predecessor of \( n \)
- Alias(n, RA) \( n \) modified by alias \( n \)
- DIRMOD(n) \( n \) modified by \( n \)

Program:
```c
int x, y, k;
R(int *b)
{
    if (*b)
    {
        b = &k;
        *b = 0;
    }
    (*b)++
}
main()
{
    R(&x);
    R(&y);
}
```

Graph:

<table>
<thead>
<tr>
<th>Reaching Alias</th>
<th>PMOD Solutions for main</th>
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<tbody>
<tr>
<td>( \phi )</td>
<td>{ x, k, y }</td>
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<td>( \phi )</td>
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<td>( \phi )</td>
<td>{ b } \n { k } \n { k }</td>
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<td>( \phi )</td>
<td>{ b } \n { k } \n { k }</td>
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<tr>
<td>( \ast b, x )</td>
<td>{ x }</td>
</tr>
<tr>
<td>( \ast b, y )</td>
<td>{ y }</td>
</tr>
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</table>
Example

Decomposition of the Analysis
MOD(n) and MOD(P)

int x, y, k;
R(int *b)
{
    if (*b)
    {
        b = &k;
        *b = 0;
    }
    (*b)++
}

main()
{
    R(&x);
    R(&y);
}

FI Alias solution for entire program

< *b, k >
< *b, x >
< *b, y >

PMOD Solution for main

{ x, y, k }

PMOD Solution for R

{ x, y, k, b }

CMOD Solutions for main

CMOD Solutions for R

<table>
<thead>
<tr>
<th>n1</th>
<th>n2</th>
<th>n3</th>
<th>n4</th>
<th>n5</th>
<th>n6</th>
<th>n7</th>
<th>n8</th>
<th>n9</th>
<th>n10</th>
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<tr>
<td>{k, x, y}</td>
<td>{k, x, y}</td>
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<td></td>
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<td></td>
<td></td>
<td></td>
<td>{ b }</td>
<td>{ k, x, y }</td>
<td>{ k, x, y }</td>
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</table>

Fig. 13. MOD₀(FIALias) solution for the example program of Figure 11.
Interprocedural Side-Effect Analysis

From Local Analysis:

- **DIRMOD(s)**: variables directly modified by assignment s (no need for dataflow analysis)
- **$B_C(\text{VarSet})$**: Translates VarSet from names in callee (F) to names in caller at call-site C

IP dataflow problem is decomposed into several dataflow equations. They are solved by iteration on the call graph.
Decomposition of the Analysis MOD(n) and MOD(P)

P – Procedure
RA – Calling Context (Reaching Aliases)
n – Program point (statement)

MOD(n) variables modified by statement n, summarizing all contexts
CMOD(n, RA) variables modified by statement n under RA
PMOD(P, RA) variables modified by procedure P, summarizing all contexts
CondLMOD(P, RA) variables modified by assignment n due to aliases after any predecessor of n
CondLIMOD(P, RA) variables modified by assignments in procedure P, under context RA

Alias Analysis in context RA
Alias(n, RA)

DIRMOD(n) variables directly modified by assignment n
Interprocedural Side-Effect Analysis

**CondLMOD(\(n, RA\)):**
variables modified by assignment \(n\) due to aliases after any predecessor of \(n\), under context \(RA\)
includes trivial aliases \(<^*p, ^*p>\) for every location.

\[
\text{CondLMOD}(n, RA) = \bigcup_{p : p \rightarrow n} \left\{ X_1 \mid (X_1, X_2) \in \text{Alias}(p, RA) \land X_2 = \text{DIRMOD}(n) \right\}
\]

**CondIMOD(P, RA):**
variables modified by assignments in procedure \(P\), under \(RA\)

\[
\text{CondIMOD}(P, RA) = \bigcup_{\text{assignments } n \in P} \text{CondLMOD}(n, RA)
\]
Interprocedural Side-Effect Analysis

$\text{PMOD}(P, RA)$:
variables modified by procedure $P$ under $RA$

$$\text{PMOD}(P, RA) = \text{condIMOD}(P, RA) \cup \bigcup_{C_Q \in P: \text{call to } Q} \text{b}_{C_Q} \left( \text{PMOD}(Q, RA') \right)$$

$RA' \in \text{contexts of}(C_Q, RA)$
Interprocedural Side-Effect Analysis

**CMOD(n,RA):**
variables modified by statement n under RA

\[
CMOD(n, RA) = \begin{cases} 
  \text{condLMOD}(n, RA) \\ 
  \bigcup_{RA' \in \text{contexts of}(n, RA)} b_n(\text{PMOD}(Q, RA')) \\
  \phi & \text{if } n \text{ is an assignment} \\
  & \text{if } n \text{ is a call to } Q \\
  & \text{otherwise}
\end{cases}
\]
Interprocedural Side-Effect Analysis

Finally:

\[ \text{MOD}(n) = \bigcup \text{CMOD}(n, RA) \]

\[ \text{MOD}(P) = \bigcup \text{PMOD}(P, RA) \]

Decomposition of the Analysis

MOD(n) and MOD(P)
Example

```c
int x, y, k;
R(int *b)
{
    if (*b)
    { b = &k;
      *b = 0;
    } (*b)++
}
main()
{
    R(&x);
    R(&y);
}
```
Example

```c
int x, y, k;
R(int *b)
{
  if (*b)
  {
    b = &k;
    *b = 0;
  }
  (*b)++
}
main()
{
  R(&x);
  R(&y);
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```

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<td>φ</td>
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<td>&lt;*b,y&gt;</td>
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INTERPROCEDURAL OPTIMIZATIONS
Inline Substitution

The code from one subroutine is substituted at the call site; formal parameters are replaced by actual parameters:

```
int f (int x) {
    int r = g(x);
    return r; }
int g(int y) {
    return 2*y}
```

- Can always be applied
- But can be too expensive (exponential blowup)
- Recompilation of a single function will cause project recompilation
Function Cloning

Specialize function for specific values of the parameters

int f(int a[], int s) {
    for (i=0;i<len(a);i++)
        a[i*s-s+1]=
        a[i*s-s+1]+3;
}

Vectorizable when s>0,
not vectorizable when s=0

• Enhances the applicability of constant propagation
Separate Compilation

The problem
Interprocedural data flow analysis introduces subtle dependences
• optimized procedures are program-specific
• correctness of object code depends on whole program
Changing one procedure can force many compilations:
• the procedure, itself, for different programs
• other procedures within those programs

Solution: Separate Compilation
• Allows subsets of a program to be compiled separately and then linked together into a final executable.
• After a module is changed, only need to re-do selected optimizations on selected procedures
• Analysis to determine which files were changed: dataflow!